

# Microwave systems

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November 2, 2002

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# Chapter 1

## Two port parameters

### 1.1 Impedance

The impedance for a lumped element may be expressed as  $V = IZ$  where  $Z$  is the impedance,  $V$  the voltage and  $I$  the current. Consequently for a 2 port network the impedance is expressed as:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (1.1)$$

for a passive network the  $Z$  parameters may be represented by the 2 port network shown in figure 1.1. The  $Z$  matrix is then given by equation 1.2.

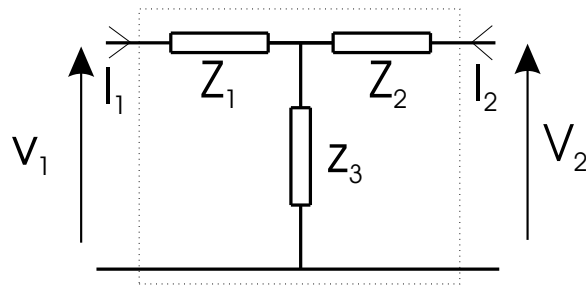


Figure 1.1: Z matrix

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_1 + Z_3 & Z_3 \\ Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (1.2)$$

**Series connection:** Two  $Z$  matrixes may be connected in series as in figure 1.1. The resulting  $Z$  matrix is simply the addition of the two matrixes as in equation 1.3.

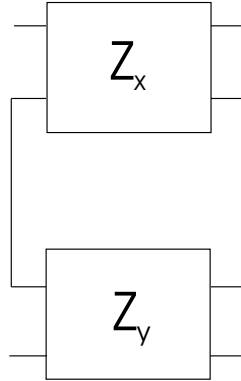


Figure 1.2: Series connection

$$[V] = ([Z_x] + [Z_y])[I] \quad (1.3)$$

## 1.2 Admittance

The admittance for a lumped element may be expressed as  $I = VY$  where  $Y$  is the admittance( $Z^{-1}$ ),  $V$  the voltage and  $I$  the current. Consequently for a 2 port network the admittance is expressed as:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (1.4)$$

for a passive network the  $Y$  parameters may be represented by the 2 port network shown in figure 1.2. The  $Z$  matrix is then given by equation 1.2.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_1 + Y_2 & -Y_2 \\ -Y_2 & Y_3 + Y_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad (1.5)$$

**Parallel connection:** Two  $Y$  matrix may be connected in parallel as in figure 1.2. The resulting  $Y$  matrix is simply the addition of the two  $Y$  matrix as shown in equation 1.6.

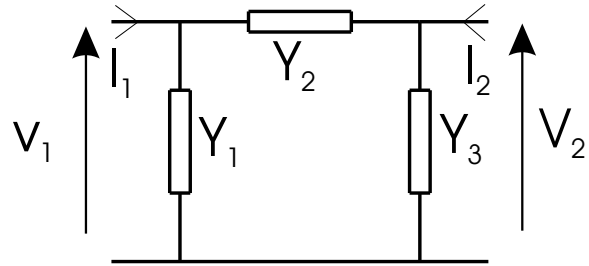


Figure 1.3: Y matrix

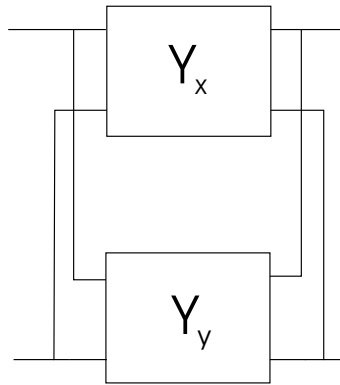


Figure 1.4: Parallel connection

$$[I] = ([Y_x] + [Y_y])[V] \quad (1.6)$$

### 1.3 ABCD matrix

An ABCD matrix used for cascading multiple system blocks. The matrix is defined as:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (1.7)$$

ABCD blocks may be cascaded as in figure 1.3. The resulting ABCD matrix is found by multiplication of the cascaded matrices as shown in equation 1.8. As matrices are not commutative, the matrixes must be multiplied in the correct order.

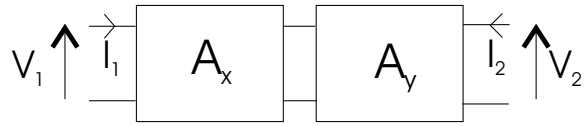


Figure 1.5: cascade connection

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_x & B_x \\ C_x & D_x \end{bmatrix} \begin{bmatrix} A_y & B_y \\ C_y & D_y \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad (1.8)$$

## 1.4 S Parameters

The previous 2 port parameters are not easily measured. Generally a two port network is characterized by S-parameters (with a network analyser), then converted to a convenient form. S-parameters are measured as the ratio of incident to reflected power. They are normalized to the reference resistance and converted to a voltage ratio.

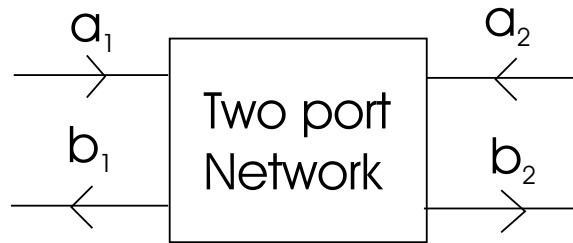


Figure 1.6: S Parameters

a is the incident voltage and b is the reflected voltage.

$$S_{11} = \frac{b_1}{a_1} \quad S_{12} = \frac{b_1}{a_2} \quad (1.9)$$

$$S_{21} = \frac{b_2}{a_1} \quad S_{22} = \frac{b_2}{a_2} \quad (1.10)$$

$$(1.11)$$

## 1.5 T-Parameters

Transmission parameters are similar to the ABCD matrix, except they are used with reflection co-efficients. They are defined as:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix} \quad (1.12)$$

T-parameters are simply converted from S-parameters:

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{S_{11}} & \frac{-S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{bmatrix} \quad (1.13)$$

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{T_{21}}{T_{11}} & T_{22} - \frac{T_{21}T_{12}}{T_{11}} \\ \frac{1}{T_{11}} & -\frac{T_{12}}{T_{11}} \end{bmatrix} \quad (1.14)$$



## Chapter 2

# Transmission Lines

Transmission lines are devices for transmitting electrical energy. The impedance of the transmission line is given by the impedance seen looking into a transmission line of infinite length. This is given by equation 2.1.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}} \left( 1 + j \left( \frac{G}{2\omega C} - \frac{R}{2\omega L} \right) \right) \quad (2.1)$$

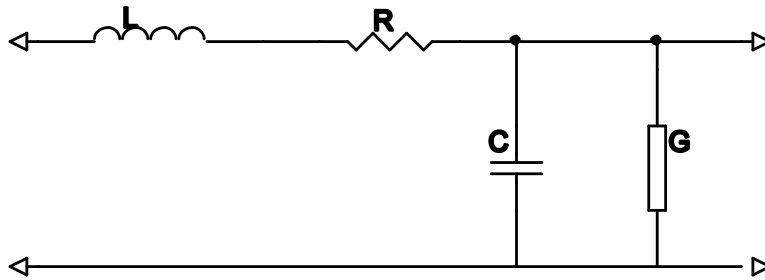


Figure 2.1: Transmission line

This is generally approximated to:

$$Z_0 = \sqrt{\frac{L}{C}} \quad (2.2)$$

If the transmission line is infinity long and one was to send a step, the line would appear to have impedance  $Z_0$ , and the waveform would propagate down

the line. If one was to shorten our line, the signal would propagate down the line, hit the end of the line and is reflected back along the line where it will add to the incident step. This line will now appear to have an infinite impedance (open circuit). If one places a short on the end of the transmission line, the signal will be reflected with opposite phase, and hence cancel at the source to look like a short. This may be verified experimentally with a long transmission line and sufficiently fast rise time.

The length of time it takes to travel the length of the line is referred to as the delay,  $\tau$ . This may be calculated by knowing the length of the line and the velocity,  $v_p$ .

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} \quad (2.3)$$

$$\tau = \frac{1}{v_p} \quad (2.4)$$

$$\beta = \frac{\omega}{v_p} \quad (2.5)$$

At low frequencies the line impedance looks like the impedance at the end of the line, however at high frequencies, the time taken to traverse from one end of the transmission line to the other may be significant such that the low frequency case no longer holds. Now one must consider it as a transmission line.

### High frequency response

If one defines  $z$  as the distance down a transmission line away from the load, then one may define the instantaneous voltage and current in the transmission line as a function of  $z$ .

$$V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z} \quad (2.6)$$

$$I(z) = \frac{V^+}{Z_0} e^{-j\beta z} - \frac{V^-}{Z_0} e^{j\beta z} \quad (2.7)$$

Where  $V^+$  is the incident wave and  $V^-$  is the reflected wave at  $z=0$ .

The amount of reflected signal is given by the reflection coefficient  $\Gamma$ . The reflection coefficient is simply the ratio of the reflected signal to the incident. At the termination the line impedance must equal the termination impedance. The difference is simply the reflected component.

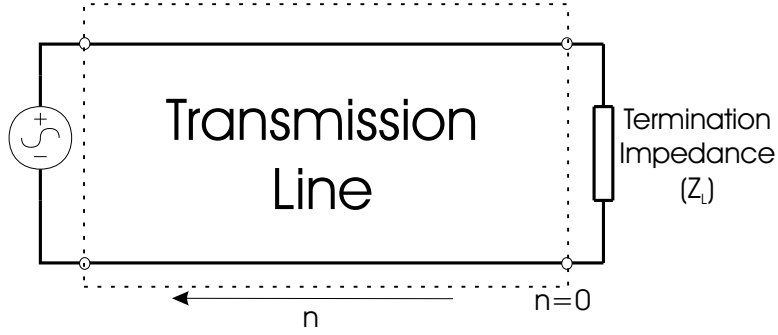


Figure 2.2: Transmission line

$$Z_L = \frac{V(z)}{I(z)} \quad (2.8)$$

$$Z_L = \frac{V^+ e^{-j\beta z} + V^- e^{j\beta z}}{\frac{V^+}{Z_0} e^{-j\beta z} - \frac{V^-}{Z_0} e^{j\beta z}} \quad (2.9)$$

$$\frac{Z_L}{Z_0} = \frac{V^+ e^{-j\beta z} + V^- e^{j\beta z}}{V^+ e^{-j\beta z} - V^- e^{j\beta z}} \quad (2.10)$$

$$\frac{Z_L}{Z_0} = \frac{1 + \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}}}{1 - \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}}} \quad (2.11)$$

$$\frac{Z_L}{Z_0} - \frac{Z_L}{Z_0} \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}} = 1 + \frac{V^- e^{j\beta z}}{V^+ e^{-j\beta z}} \quad (2.12)$$

$$\frac{Z_L}{Z_0} - 1 = \frac{V^- e^{2j\beta z}}{V^+} + \frac{V^- e^{2j\beta z}}{V^+} \frac{Z_L}{Z_0} \quad (2.13)$$

$$\frac{Z_L - Z_0}{Z_0} = \frac{V^-}{V^+} e^{2j\beta z} \left( \frac{Z_L}{Z_0} + 1 \right) \quad (2.14)$$

$$\frac{Z_L - Z_0}{Z_L + Z_0} e^{-2j\beta z} = \frac{V^-}{V^+} \quad (2.15)$$

$$(2.16)$$

Therefore:

$$\Gamma(z) = \frac{V^-(z)}{V^+(z)} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2j\beta z} \quad (2.17)$$

Given the length and the frequency, one may then calculate the impedance along the line.

$$Z(z) = \frac{V(z)}{I(z)} = \frac{V^+ + V^-}{\frac{V^+}{Z_0} - \frac{V^-}{Z_0}} \quad (2.18)$$

$$= \frac{V^+ + V^-}{V^+ - V^-} Z_0 \quad (2.19)$$

$$= \frac{V^+ + V^+ \Gamma}{V^+ - V^+ \Gamma} Z_0 \quad (2.20)$$

$$= \frac{1 + \Gamma}{1 - \Gamma} Z_0 \quad (2.21)$$

**Other useful equations [2]**

$$\text{Mismatch loss} = \frac{P_i}{P_t} = 10 \log \frac{1}{1 - |\Gamma_L|^2} \quad (2.22)$$

$$\text{Return loss} = \frac{P_i}{P_r} = 10 \log \frac{1}{|\Gamma_L|^2} = -20 \log |\Gamma_L| \quad (2.23)$$

### **Voltage Standing Wave Ratio**

The voltage standing wave ratio (VSWR) is the ratio of the maximum voltage to the minimum voltage.

$$V_{max} = \max(V^+ + V^-) \quad (2.24)$$

$$V_{min} = \min(V^+ - V^-) \quad (2.25)$$

$$VWSR = \frac{V_{max}}{V_{min}} \quad (2.26)$$

$$= \frac{\max(V^+ + |\Gamma|V^+)}{\min(V^+ - |\Gamma|V^+)} \quad (2.27)$$

$$= \frac{V^+ \max(1 + |\Gamma|)}{V^+ \min(1 - |\Gamma|)} \quad (2.28)$$

$$= \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (2.29)$$

Therefore:

$$|\Gamma| = \frac{VWSR - 1}{VWSR + 1} \quad (2.30)$$

$$\text{Max impedance} = Z_0 \cdot VWSR \quad (2.31)$$

$$\text{Min impedance} = \frac{Z_0}{VWSR} \quad (2.32)$$

Therefore one may change the impedance by selecting a suitable line length.

## Chapter 3

# Amplifiers

### 3.1 Stability

An amplifier is stable if  $|\Gamma_{in}| < 1$  and  $|\Gamma_{out}| < 1$ . Suitable source and load impedances may be determined from a particular device with given S-parameters.

**Unconditional stability:** An amplifier is said to be "unconditionally stable" where it is stable for any load and source impedance. This may be determined from the Rollet Stability Factor:

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} \quad (3.1)$$

where:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} \quad (3.2)$$

Amplifier unconditionally stable for:

$$K > 1 \quad (3.3)$$

$$|\Delta| < 1 \quad (3.4)$$

If the amplifier is not unconditionally stable, then it may be be conditionally stable. An amplifier is conditionally stable if it is stable for a limited set of Source and load impedances. These may be determined from stability circles.

These are given for the load and source as:

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad (3.5)$$

$$R_L = \left| \frac{S_{12}S_{21}}{|S_{22}| - |\Delta|^2} \right| \quad (3.6)$$

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad (3.7)$$

$$R_S = \left| \frac{S_{12}S_{21}}{|S_{11}| - |\Delta|^2} \right| \quad (3.8)$$

Where  $C_L$  and  $R_L$  are the center and radius respectively for the load.  $C_S$  and  $R_S$  is the center and radius respectively for source. \* denotes the complex conjugate.

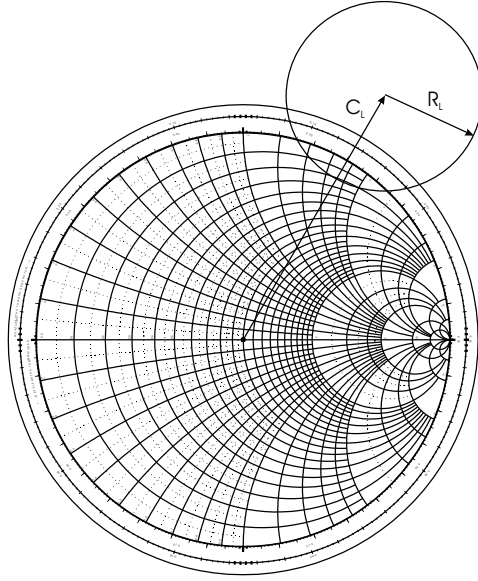


Figure 3.1: Stability Circle

For input:

If  $|S_{11}| < 1$  then the center of the smith chart is stable.

If  $|S_{11}| > 1$  then the center of the smith chart is unstable.

If the stability circle encompasses the center then it takes on the stability of the center. If the stability circle is outside the center as in figure 3.1. Then the stability circle represents the region that has opposite context to the center.

For example, if  $|S_{11}| < 1$  and the system has input stability circle as in figure 3.1. The the system will be stable for any region inside the smith chart but outside the stability circle.

**additional information** <http://images.rfdesign.com/files/4/0102Misra38.pdf>  
<http://www.electronicworkbench.com/pdf/s-param.pdf>

## 3.2 Gain

The gain of a system is given by the transducer gain which is the ratio of power delivered to load and power available from the source [3].

$$G_T = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S - S_{22}\Gamma_L + \Delta\Gamma_S\Gamma_L|^2} \quad (3.9)$$

If the device is unilateral then by definition  $S_{12} = 0$  and we can simplify  $G_T$  [1].

$$G_{TU} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S - S_{22}\Gamma_L + S_{11}S_{22}\Gamma_S\Gamma_L|^2} \quad (3.10)$$

$$G_{TU} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S - S_{22}\Gamma_L(1 - S_{11}\Gamma_S)|^2} \quad (3.11)$$

$$G_{TU} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|(1 - S_{11}\Gamma_S)(1 - S_{11}\Gamma_S)|^2} \quad (3.12)$$

$$G_{TU} = \frac{|S_{21}|^2(1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{11}\Gamma_S|^2|1 - S_{11}\Gamma_S|^2} \quad (3.13)$$

This is convenient as we can now split the function into 3 independent gains, namely:  $G_S$ ,  $G_oO$ ,  $G_L$ . Where  $G_{TU} = G_S \cdot G_O \cdot G_L$

$$G_S = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} \quad (3.14)$$

$$G_O = |S_{21}|^2 \quad (3.15)$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \quad (3.16)$$

When to assume the device is unilateral is given by the unilateral figure of merit, U [1].  $U < 0.1dB$  justifies a reasonable assumption.



$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} \quad (3.17)$$

### 3.2.1 design

One may want the maximum gain. The highest gain will be when the input and output are matched to the source and load impedance respectively.

Therefore:

$$\begin{aligned} \Gamma_S &= (S_{11})^* \\ \Gamma_L &= (S_{22})^* \end{aligned}$$

Thus for maximum gain(Input/output matching):

$$G_S = \frac{1}{1 - |S_{11}|^2} \quad (3.18)$$

$$G_O = |S_{21}|^2 \quad (3.19)$$

$$G_L = \frac{1}{1 - |S_{22}|^2} \quad (3.20)$$

If one wanted to reduce the gain, this may be done by de-tuning the matching network. The change in gain may be represented by constant gain circles on the smith chart.

For the Unilateral case [1]:

$$g_S = \frac{G_S}{G_{max}} = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} (1 - |S_{11}|^2) \quad (3.21)$$

$$g_L = \frac{G_L}{G_{max}} = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} (1 - |S_{22}|^2) \quad (3.22)$$

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2} \quad (3.23)$$

$$R_S = \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2} \quad (3.24)$$

$$C_L = \frac{g_L S_{22}^*}{1 - (1 - g_L)|S_{22}|^2} \quad (3.25)$$

$$R_L = \frac{\sqrt{1 - g_L}(1 - |S_{22}|^2)}{1 - (1 - g_L)|S_{22}|^2} \quad (3.26)$$

### 3.3 Noise Performance

The noise figure is given by

$$F = F_{min} + \frac{4R_N}{Z_0} \frac{|\Gamma_S - \Gamma_{opt}|^2}{(1 - |\Gamma_S|^2)|1 + \Gamma_{opt}|} \quad (3.27)$$

Where:

$\Gamma_S$  - source reflection coefficient

$\Gamma_{opt}$  - optimum source reflection coefficient

$F_{min}$  - minimum noise figure of transistor ( $\Gamma_s = \Gamma_{opt}$ )

$R_N$  - equivalent noise resistance of transistor

$Z_0$  - characteristic impedance to which circuit is referenced

The noise figure becomes a function of source impedance. There may exist conflicting interest, such as low noise and high gain. Thus one may define noise figure circles and chose an impedance which best satisfies both constraints. The center( $C_F$ ) and radius( $R_F$ ) for these noise figure circles are defined in equation 3.28 and 3.29 respectively.

$$C_F = \frac{\Gamma_{opt}}{N + 1} \quad (3.28)$$

$$R_F = \frac{\sqrt{N^2 + N(1 - |\Gamma_{opt}|^2)}}{N + 1} \quad (3.29)$$

$$N = \frac{F - F_{min}}{4R_N/Z_0} |1 + \Gamma_{opt}|^2 \quad (3.30)$$

**other formula**

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \quad (3.31)$$

#### 3.3.1 additional information

<http://www.sandiego.edu/~ekim/e194rfs01/lec23ek.pdf>

## Chapter 4

# Microwave Oscillators

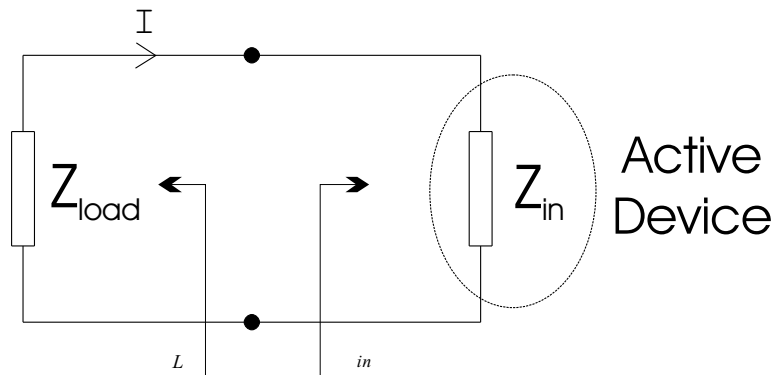


Figure 4.1: Oscillator Design

Figure 4 shows a basic oscillator design. It consists of some active component ( $Z_{in}$ ) and a load ( $Z_{load}$ ). For oscillations to occur, the current  $I$  must be non-zero. Thus using Kirchoff's law:

$$(Z_{in} + Z_{load})I = 0 \quad (4.1)$$

$$Z_{load} = -Z_{in} \quad (4.2)$$

$$R_L + R_{in} = 0 \quad (4.3)$$

$$X_L + X_{in} = 0 \quad (4.4)$$

Thus given some active element ( $Z_{in}$ ) one simply selects a lumped element to satisfy equation 4.2 at the required oscillation frequency.

For stable operation, the active device must be non-linear, such that [1]:

$$\frac{\partial R_{in}}{\partial I} \frac{\partial (X_L + X_{in})}{\partial \omega} - \frac{\partial X_{in}}{\partial I} \frac{\partial R_{in}}{\partial \omega} > 0 \quad (4.5)$$

## 4.1 Transistor Oscillators

One may use a negative resistance diode or similar active component for  $Z_{in}$ . An unstable transistor may also be used. One may cause a transistor to be unstable by selecting an appropriate terminating impedance ( $Z_T$ ) for the output of the transistor and/or including positive feedback.

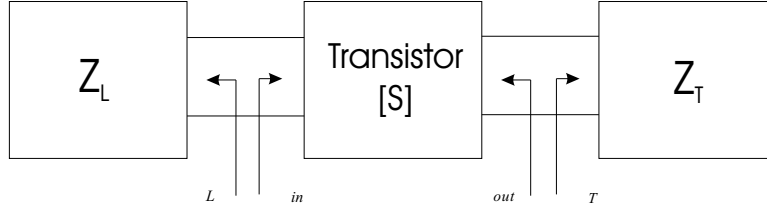


Figure 4.2: Transistor Oscillator

Thus one can derive an equation for  $\Gamma_{in}$ . The Impedance  $Z_T$  is transformed by the 2-port transistor network to give:

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{22}\Gamma_T} \quad (4.6)$$

Given that the system is unstable, we can simply choose  $Z_L$  to cause oscillations by equation 4.2

This design method assumes small signals (by virtue to S-parameters). However the amplitude of oscillation may be sufficiently large to void this assumption. As the power level increases,  $R_{in}$  will increase. To compensate for this, it is advisable to reduce the value of  $R_L$ . Reference [1] recommends:

$$R_L = \frac{-R_{in}}{3} \quad (4.7)$$

### 4.1.1 Feedback

One may increase the instability of the transistor by introducing positive feedback. A simple method for analyzing feedback is to take the 2-port transistor network, add the feedback and convert back to a 2-port network. Thus one may use the simple analysis presented previously.

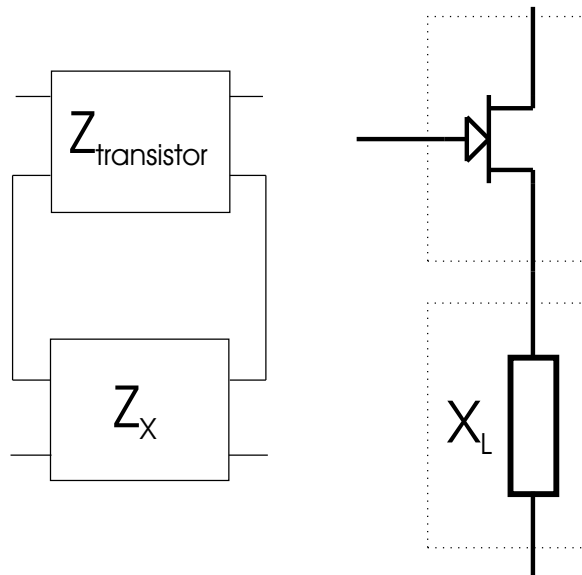


Figure 4.3: Series Z-parameter

If one takes a common source amplifier example, one may alter the characteristics of the transistor by adding a reactive lumped component to the source, as shown in figure 4.1.1. The S-parameter for the transistor is converted to a Z-matrix such that one may add the series reactance. The corresponding S-parameter matrix is determined by first computing the new Z-matrix and converting to an S-matrix.

$$Z = Z_{transistor} + Z_X \quad (4.8)$$

### 4.1.2 Resonators

The oscillation frequency is determined by matching the Active device impedance to the load impedance at the oscillation frequency of interest. If the matching network is not particularly frequency selective, low Q. Then the frequency of

oscillation will tend to drift, creating phase noise. This will introduce new, typically unwanted frequency components.

To increase the selectivity of the oscillator, a resonator may be used. A typical resonator is the dielectric resonator. It is a ceramic material that is coupled to the circuit by placed it close to a transmission line. It may be modelled as in figure 4.1.2. Where the coupling is modelled as a transformer with turns ratio  $N$ .

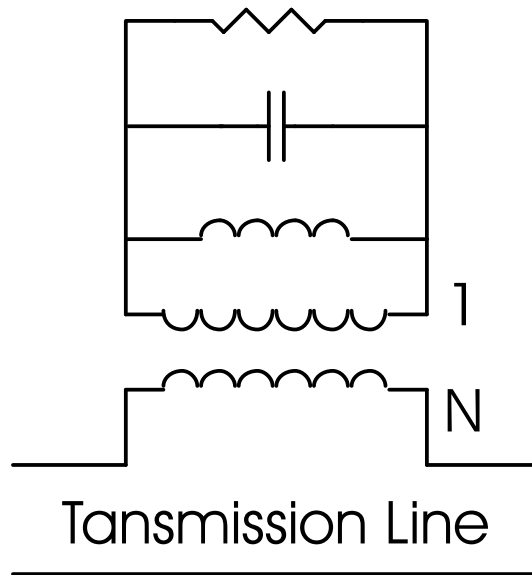


Figure 4.4: Resonator Model

Figure 4.1.2 shows the implementation of such a resonator. The resonator is coupled to the transmission line by placing it some distance away. Its impedance at its resonant frequency is given by its resistance and equivalent turns ratio,  $N^2R$

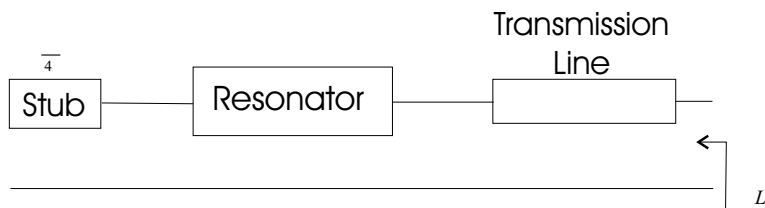


Figure 4.5: Resonator Implementation

This example is implemented with a  $\frac{\lambda}{4}$  stub. This appears as a short circuit at the frequency of interest and aids in coupling the transmission line to the resonator. Thus one may determine the reflection coefficient:

$$\Gamma_L = \left( \frac{N^2 R - Z_0}{N^2 R + Z_0} \right) e^{-2j\beta l} \quad (4.9)$$

# Chapter 5

## Noise

### 5.1 Noise figure

The noise factor and corresponding noise figure is given by:

$$\text{Noise Factor } F = \frac{SNR_{out}}{SNR_{in}} \quad (5.1)$$

$$\text{Noise Figure } NF = 10 \log F \quad (5.2)$$

For an amplifier the noise factor may be calculated from the amplifier noise sources,  $E_n$  and  $I_n$  and source resistance  $R_s$  such that:

$$F = \frac{4kTR_s + E_n^2 + I_n^2 R_s^2}{4kTR_s} \quad (5.3)$$

It may be subsequently found that the optimal source resistance  $R_s$  is given by:

$$R_s = \frac{E_n}{I_n} \quad (5.4)$$



### 5.1.1 Cascaded systems

In a cascaded system noise dominated by first stage. Thus ensure input stage has low noise figure.

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \frac{F_4 - 1}{G_1 G_2 G_3} + \dots \quad (5.5)$$

where G is the amplifier gain and F the noise figure. An equivalent method is to use noise temperature.

$$T = 290(10^{NF/10} - 1) \quad (5.6)$$

$$T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots \quad (5.7)$$

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