# Device Modeling

Peter Blockley

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# Contents

# Chapter 1

# Semi-conductor Physics

## 1.1 Intrinsic semiconductor

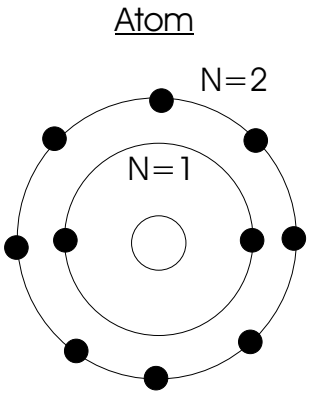


Figure 1.1: Atom Model

Each atom contains a number of energy states that may be filled by electrons. Electrons like to occupy the lowest energy state, such that the lower states are filled first, followed by the higher energy states.

In each energy state there may exist a number of electrons. The first state contains 2 electrons, the 2nd state 8 electrons, ect.

In a crystal each electrons "sees" the entire latus. Therefore the available electrons will fill the available energy bands of the crystal filling the lower energy band before the higher ones.

All the available electrons fill each energy band progressively until all the electrons are occupying energy states. Once an energy band is full, the net velocity of electrons is 0. As one electron moves in one direction, another must be moving in the opposite direction. If the number of available electrons completely fills all available energy bands with few left over, the material is an insulator. The distinction of a semi-conductor comes about if the energy required to go to the next available band is small.

A conductor is one in which the available electrons only partially fill the energy band. This band is then called a conduction band. An energy band which is completely filled with electrons is called a valence band.

The fermi-level is the energy level at which there is an equal probability of an electron existing above it or below it. When the valence band is full and the conduction band is empty, It is equally likely that an electron will exist in the conduction or valence band. Hence it resides in the middle of the forbidden band for an intrinsic semi-conductor.

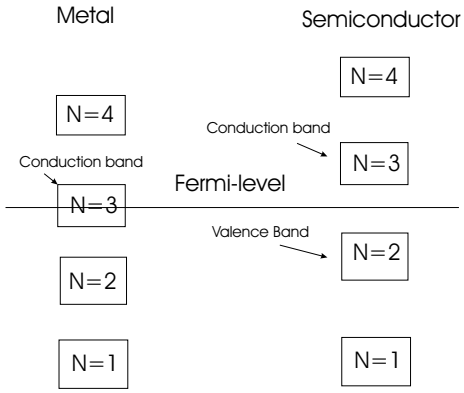


Figure 1.2: Fermi Level

If there are no electrons in the conduction band of a semi-conductor it won't conduct. To move electrons out of the valence band and into the conduction band, one needs to give them energy. This may be though heat, incident light or high electric field. As most semiconductors operate at non-zero temperature, there is generally some electrons in the conduction band. This also means that if the semi-conductor get too hot(125 degrees C) for silicon, excess electrons will exist in the conduction band, hence the semi-conductor will act more like a conductor.

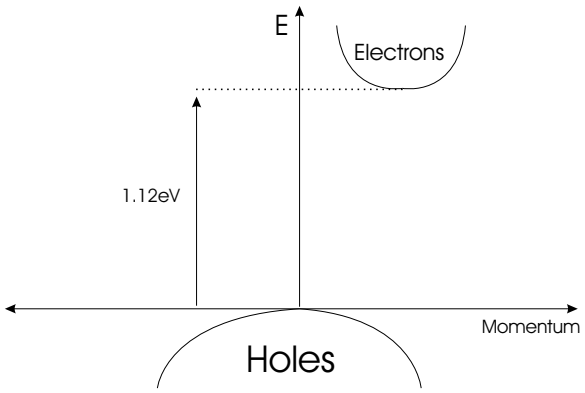


Figure 1.3: si Band Gap

### 1.1.1 Band-gap

The total energy of an electron is given by its momentum and its potential energy. To move an electron from the conduction band to the valence band, it may need to undergo a change in potential energy and a change in momentum. There are two basic material types, in-direct and direct band gap materials. In a indirect bang gap material, such as silicon, shown in figure 1.3. To move into the valence band, the electron must undergo a change in momentum and energy [1]. The chance of this event is small. Typically this process is achieved in several steps. The electron will first move to a trap site in the forbidden band before moving into the valence band. A change in potential energy will result in the release of a photon, while a change in momentum will produce a phonon. A phonon being a mechanical vibration which heats the crystal lattice.

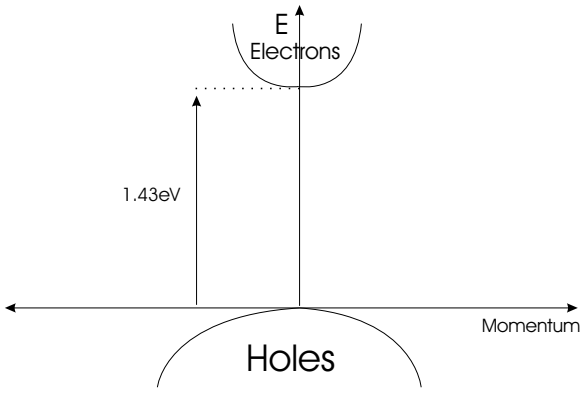


Figure 1.4: GaAs Band Gap

In a direct band-gap material such as GaAs, only a change in energy is required, as seen in figure 1.4. As such GaAs is very efficient at producing light, although in the infrared spectrum.

## 1.2 Extrinsic semiconductor

One may also dope the semiconductor material. Semi-conductor materials are doped with impurities chosen to give the material special characteristics. One may want to add extra electrons or remove electrons.

Doping atoms are chosen from elements in group III or V of the periodic table[1] which are similar in size to silicon atoms. Thus individual intrinsic semiconductor atoms may be replaced with dopant atoms to form an extrinsic semi-conductor.

The binding energy of the outer electron added by the impurity is weak. This is represented by placing the excess electrons just below the conduction band. Thus very little energy is required to move these electrons into the conduction band. Thus an extrinsic semi-conductor operating at room temperature will have most of these "extra" electrons existing in the conduction band. Thus at normal operating temperature, $n_c \approx N_d$. Where $n_c$ is number of conduction electrons and $N_d$ is the number of dopant atoms.

### 1.2.1 Fermi Level

The number of energy levels in the conduction band is given by[7] $N_c$:

$$N_c \approx 2\left(\frac{2m_e\pi kT}{h^3}\right)^{\frac{3}{2}} \tag{1.1}$$

where $m_e$ is the effective mass of an electron.

The number of energy levels in the valence band is given by $N_v$

$$N_v \approx 2\left(\frac{2m_h\pi kT}{h^3}\right)^{\frac{3}{2}} \tag{1.2}$$

where $m_h$ is the effective mass of a hole.

For an intrinsic semi-conductor, the number of conduction electrons must equal the number of conduction holes. Such that

$$n_c = n_v \tag{1.3}$$

where $n_c$ is the number electrons in the conduction band and $n_v$ is the number of conduction holes in the valence band, given by:

$$n_c = N_c e^{\frac{E_f - E_c}{kT}} \tag{1.4}$$

$$n_v = N_v e^{\frac{E_f - E_v}{kT}} \tag{1.5}$$

for an n-type extrinsic semiconductor, the number of conduction electrons $n_c$ must equal the number of conduction holes plus the number of ionized donor atoms, $n_d$.

$$n_c = n_v + n_d \tag{1.6}$$

where:

$$n_d \approx N_d e^{\frac{E_f - E_d}{kT}} \tag{1.7}$$

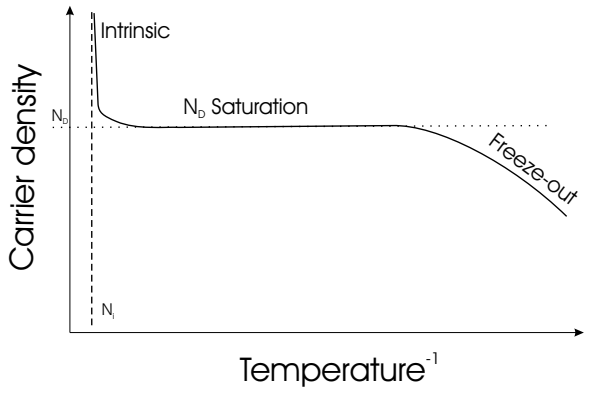


Figure 1.5: Extrinsic Conductivity

Figure 1.5 shows this relationship against temperature. At the operating temperature, the electrons available for conduction is relatively constant, as most donor electrons exist in the conduction band. For high temperatures electrons from the valance band begin to populate the conduction band, significantly increasing the carrier density. The electrons in the conduction band are now

dominated by electrons from the intrinsic semiconductor and it is said to be intrinsic. For very low temperatures, the donor electrons no longer populate the conduction band and the semi-conductor is said to freeze out.

## 1.3 conduction

**electron mobility:** When an electric field is applied to a semiconductor, the electrons experience a force and are accelerated in the opposite direction of the electric field. This acceleration is inhibited by what we term 'collisions' [1]. When a collision occurs, the velocity of the electron drops to zero and it accelerates again. The average time between collisions is given by $\tau_c$.

The effect is a constant drift velocity $V_n$ given by:

$$V_n = -\mu \cdot \xi \tag{1.8}$$

$$\mu = \frac{q\tau_c}{m_e} \tag{1.9}$$

where $\mu$ is the mobility. Its derivation is complicated as the velocities have a Maxwellian distribution.

The current density $J_n$ is given by:

$$J_n = \frac{I}{A} = -qnV_n \tag{1.10}$$

where $n$ is the number of electrons per unit area $A$ and q their charge. One may also express the current density in terms of the conductivity $\sigma$:

$$J_n = \sigma\xi \tag{1.11}$$

$$\sigma = qn\mu \tag{1.12}$$

where $\sigma$ is the conductivity in $\frac{S}{m}$ and $\xi$ the electric field.

Conduction is further complicated by additional diffusion of carriers caused by space charge. The voltage drop across the semiconductor is gradual and therefore sets up an electron density gradient. Electrons which exist at higher densities experience a force towards less dense region. Thus a Diffusion coefficient $D_n$ is defined along with electron density gradient $\nabla n$.

$$J_n = qn\mu_n\xi + qD_n\nabla n \tag{1.13}$$

where

$$D_n = \frac{kT}{q}\mu_n \tag{1.14}$$

## 1.4 Breakdown

# Chapter 2

# Diode

Diodes junctions may be formed with N type and P type semi-conductor or semi-conductor and Metal junction.

We will begin our analysis by defining the "work function" which is the minimum amount of energy required for an electron to overcome the binding energy and escape the conductor into free space. The reference is taken from the fermi level and is given as $\Phi$.

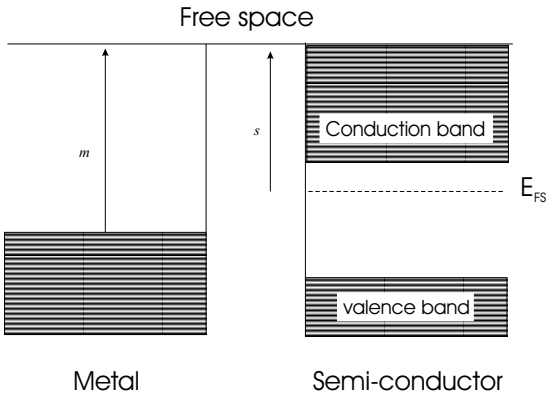


Figure 2.1: none

When the Metal and Semi-conductor come together, excess electrons diffuse from the semi-conductor to the metal until the fermi levels are equal. As electrons diffuse from the semi-conductor to the metal, they leave behind their donor atom. This depletes the region near the interface of charge carriers and hence this region does not conduct well. Thus for charges to diffuse across this region,

they require additional energy. Since the extent of the depletion of charges is some function of distance, the conduction band is bent to represent the energy required for diffusion in that region.

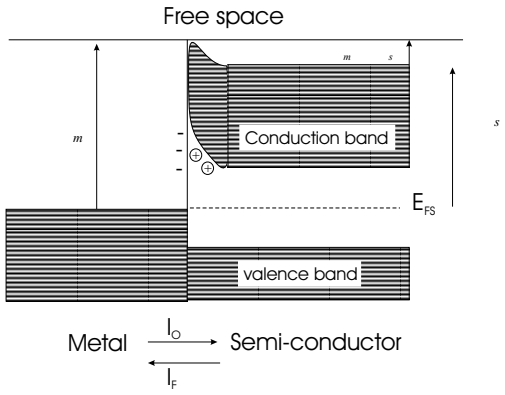


Figure 2.2: none

There will now exist 2 currents. Electrons diffuse from metal to semi-conductor over the barrier induced by the depletion region $I_O$. Electrons diffuse from the semi-conductor over this barrier in the opposite direction $I_F$. When there is no external applied voltage, there current will be equal and opposite.

Electrons will diffuse from semi-conductor to metal if they have enough energy to overcome the barrier. The energy of an electron is statistical, and is given by the Maxwell-Boltzmann distribution [1]. Thus as the temperature increases, the probability of the number of electrons occupying higher energy states increases. For low doping densities, the Maxwell-Boltzmann distribution may also be applied. However for non-zero temperature most excess electrons from the dopant will exist in the conduction band.

Maxwell-Boltzman distribution says: given an energy W, the number of electrons with energy greater than W is given by:

$$n_W = n \cdot e^{\frac{-W}{kT}} \tag{2.1}$$

where n is the number density of electrons, k Boltzmann's constant and T the temperature.

The Maxwell-Boltzmann distribution does not apply to metals. The reason may be found in the referenced [1]. For small voltages one may assume $I_O$ to be constant.

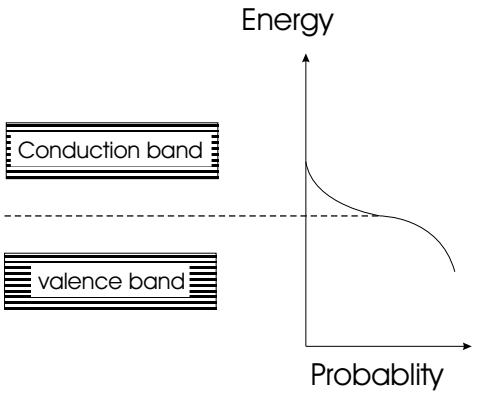


Figure 2.3: none

$$I_F = nqe^{\frac{-q\Psi}{kT}} \tag{2.2}$$

$I_O = I_F$ for no external potential. This equilibrium exists for a diffusion potential we will call $\Psi$, where $\Psi = \phi_m - \phi_s$. Thus the energy of an electron to cross this barrier is given by $q\Psi$.

$$I_O = I_F = nq \cdot e^{\frac{-q\Psi}{kT}} \tag{2.3}$$

$$nq = I_O \cdot e^{\frac{q\Psi}{kT}} \tag{2.4}$$

The fermi level of the n-type semi-conductor may be increased by an external potential. This will increase the number of electrons with energy to diffuse across the barrier. Thus the energy for electron to cross the barrier will become $q(\Psi - V)$.

$$I_F = nq \cdot e^{\frac{-q(\Psi - V)}{kT}} \tag{2.5}$$

$$I_F = I_O \cdot e^{\frac{q\Psi}{kT}} \cdot e^{\frac{-q(\Psi - V)}{kT}} \tag{2.6}$$

$$I_F = I_O \cdot e^{\frac{qV}{kT}} \tag{2.7}$$

$$\tag{2.8}$$

$$\text{Thus } I_D = I_F - I_O \tag{2.9}$$

$$\text{Let } V_T = \frac{kT}{q} \tag{2.10}$$

Then:

$$I_D = I_O\left(e^{\frac{V_D}{V_T}}\right) - I_O \tag{2.11}$$

$$I_D = I_O\left(e^{\frac{V_D}{V_T}} - 1\right) \tag{2.12}$$

Some examples of this effect may be seen at the website:

http://jas2.eng.buffalo.edu/

## 2.1 depletion Width

The depletion width is given by the required amount of charge to be displaced to realize the diffusion potential. Thus if one knows the doping density and relative permittivity of the material, then one may calculate the depletion width.

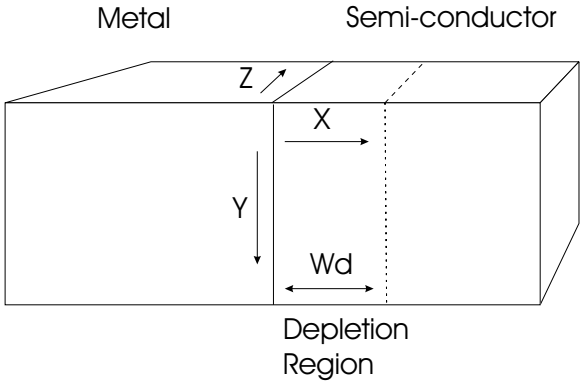


Figure 2.4: none

The Depletion width is easiest realized by Poisson's Equation [1]. Assume constant doping density.

By Poisson's equation:

$$\frac{\partial^2}{\partial x^2}V(x) = \frac{\rho}{\varepsilon} \tag{2.13}$$

where $\rho = qN_d$ the charge density, $\varepsilon$ the permittivity and x the distance from the junction.

$$V = \frac{qN_d x^2}{2\varepsilon_0 \varepsilon_r} \tag{2.14}$$

The depletion width is some function of barrier height, $(\Psi - V)$. Therefore solve for $x = W_d$

$$\Psi - V = \frac{qN_d W_d^2}{2\varepsilon_0 \varepsilon_r} \tag{2.15}$$

$$W_d = \sqrt{\frac{2\varepsilon_0 \varepsilon_r (\Psi - V)}{qN_d}} \tag{2.16}$$

## 2.2 Depletion capacitance

Capacitance is the change in charge for change in voltage over the depletion width.

Given:

$$C = \frac{\partial Q}{\partial V} \tag{2.17}$$

$$Q = qN_d W_d Y Z \tag{2.18}$$

Then:

$$C = \frac{\partial (qN_d W_d Y Z)}{\partial (\Psi - V)} \tag{2.19}$$

$$C = qN_d Y Z \frac{\partial W_d}{\partial (\Psi - V)} \tag{2.20}$$

$$C = qN_d Y Z \cdot \frac{2\varepsilon_0 \varepsilon_r}{qN_d} \cdot -\frac{1}{2} \left( \frac{2\varepsilon_0 \varepsilon_r (\Psi - V)}{qN_d} \right)^{-\frac{1}{2}} \tag{2.21}$$

$$C = -\frac{1}{2} qN_d Y Z \cdot \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{qN_d}} \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{qN_d}} \cdot \sqrt{\frac{qN_d}{2\varepsilon_0 \varepsilon_r (\Psi - V)}} \cdot \frac{\sqrt{\Psi}}{\sqrt{\Psi}} \tag{2.22}$$

$$C = -\frac{1}{2} qN_d Y Z \cdot \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{qN_d}} \cdot \frac{1}{\sqrt{\Psi}} \cdot \sqrt{\frac{\Psi}{\Psi - V}} \tag{2.23}$$

$$C = -\frac{1}{2} qN_d Y Z \cdot \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{qN_d \Psi}} \cdot \frac{1}{\sqrt{1 - V/\Psi}} \tag{2.24}$$

$$\text{Let } C_0 = -\frac{1}{2} q N_d Y Z \cdot \sqrt{\frac{2\varepsilon_0 \varepsilon_r}{q N_d \Psi}} \tag{2.25}$$

$$\text{Therefore } C = \frac{C_0}{\sqrt{1 - V/\Psi}} \tag{2.26}$$

## 2.3 C-V profiling

From [7]

Per unit area:

$$C^2 = \frac{1}{4} q^2 N_d^2 \frac{2\varepsilon_0 \varepsilon_r}{q N_d \Psi} \cdot \frac{1}{1 - V/\Psi} \cdot \frac{\Psi}{\Psi} \tag{2.27}$$

$$C^2 = \frac{q N_d \varepsilon_0 \varepsilon_r}{2(\Psi - V)} \tag{2.28}$$

$$\frac{1}{C^2} = \frac{2}{q N_d \varepsilon_0 \varepsilon_r} (\Psi - V) \tag{2.29}$$

$$\frac{\partial\left(\frac{1}{C^2}\right)}{\partial V} = \frac{2}{q N_d \varepsilon_0 \varepsilon_r} \tag{2.30}$$

# Chapter 3

# Mesfet

Assume an N channel MESFET with uniform doping and sharp depletion region shown in figure 3.1.

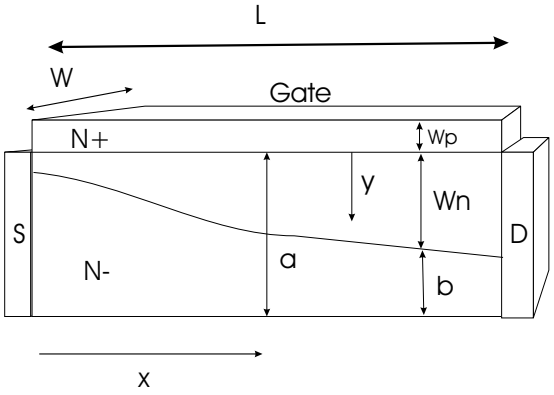


Figure 3.1: MESFET

The depletion region $W_n$ is given by the depletion width for a diode. Where the voltage is the voltage from the gate to the channel, where the channel voltage is given for a position x along the channel as $V_{gc}(x)$.

$$W_n(x) = \sqrt{\frac{2\varepsilon_0\varepsilon_r(\Psi - V_{gc}(x))}{qN_d}} \tag{3.1}$$

$$W_n(x)^2 = \frac{2\varepsilon_0\varepsilon_r(\Psi - V_{gc}(x))}{qN_d} \tag{3.2}$$

$$\frac{W_n(x)^2 qN_d}{2\varepsilon_0\varepsilon_r} = \Psi - V_{gc}(x) \tag{3.3}$$

$$V_{gc}(x) = \Psi - \frac{W_n(x)^2 qN_d}{2\varepsilon_0\varepsilon_r} \tag{3.4}$$

$$\frac{dV_{gc}(x)}{dW_n(x)} = -\frac{2W_n(x)qN_d}{2\varepsilon_0\varepsilon_r} \tag{3.5}$$

The current density in the channel is given by:

$$J_n = \sigma\xi \tag{3.6}$$

$$I_n(x) = \sigma\xi \cdot W \cdot b(x) \tag{3.7}$$

$$I_n(x) = -\sigma\frac{dV_{gc}(x)}{dx}W(a - W_n(x)) \tag{3.8}$$

where

$$\xi = -\frac{dV_{gc}(x)}{dx} \tag{3.9}$$

Therefore

$$I_n(x) = -\sigma aW\left(1 - \frac{W_n(x)}{a}\right)\frac{dV_{gc}(x)}{dWn(x)}\frac{dWn(x)}{dx} \tag{3.10}$$

$$\int_0^L I_n(x)\,dx = \int_0^L -\sigma aW\left(1 - \frac{W_n(x)}{a}\right)\frac{dV_{gc}(x)}{dW_n(x)}\frac{dW_n(x)}{dx}\,dx \tag{3.11}$$

$$I_n \cdot L = -\sigma aW\int_{Wn(0)}^{W_n(L)}\left(1 - \frac{W_n(x)}{a}\right)\frac{dV_{gc}(x)}{dW_n(x)}\,dW_n(x) \tag{3.12}$$

$$\tag{3.13}$$

substitute from equation 3.5

$$I_n = \frac{-\sigma a W}{L} \int_{W_n(0)}^{W_n(L)} \left(1 - \frac{W_n(x)}{a}\right)\left(-\frac{2W_n(x)qN_d}{2\varepsilon_0\varepsilon_r}\right) dWn(x) \tag{3.14}$$

$$I_n = \frac{\sigma a W 2qN_d}{2\varepsilon_0\varepsilon_r L} \int_{W_n(0)}^{W_n(L)} \left(W_n(x) - \frac{W_n(x)^2}{a}\right) dWn(x) \tag{3.15}$$

$$I_n = \frac{2\sigma a W qN_d}{2\varepsilon_0\varepsilon_r L} \left[\frac{W_n^2(x)}{2} - \frac{W_n^3(x)}{3a}\right]_{W_n(0)}^{W_n(L)} \tag{3.16}$$

$$I_n = \frac{2\sigma a W qN_d}{2\varepsilon_0\varepsilon_r L} \left[\frac{W_n^2(L) - W_n^2(0)}{2} - \frac{W_n^3(L) - W_n^3(0)}{3a}\right] \tag{3.17}$$

$$I_n = \frac{2\sigma a W qN_d a^2}{6L \cdot 2\varepsilon_0\varepsilon_r} \left[\frac{3(W_n^2(L) - W_n^2(0))}{a^2} - \frac{2(W_n^3(L) - W_n^3(0))}{a^3}\right] \tag{3.18}$$

One defines constant $\beta$ as the channel conductance with no depletion. And the work function to deplete the channel $W_{00}$[4]:

$$W_{00} = \Psi - V_{to} = \frac{qN_d a^2}{2\varepsilon_0\varepsilon_r} \tag{3.19}$$

$$\beta = \frac{\sigma a}{3LW_{00}} \tag{3.20}$$

We now define $V_{to}$, the voltage such that the channel is pinched off. $d$ is the ratio of channel depletion to maximum depletion for the drain. $s$ the ratio of channel depletion to maximum depletion for the source.

$$d = \frac{W_n(L)}{a} = \frac{\sqrt{\frac{2\varepsilon_0\varepsilon_r(\Psi - V_{gd})}{qN_d}}}{\sqrt{\frac{2\varepsilon_0\varepsilon_r(\Psi - V_{to})}{qN_d}}} = \sqrt{\frac{\Psi - V_{gd}}{W_{00}}} \tag{3.21}$$

$$s = \frac{W_n(0)}{a} = \frac{\sqrt{\frac{2\varepsilon_0\varepsilon_r(\Psi - V_{gs})}{qN_d}}}{\sqrt{\frac{2\varepsilon_0\varepsilon_r(\Psi - V_{to})}{qN_d}}} = \sqrt{\frac{\Psi - V_{gs}}{W_{00}}} \tag{3.22}$$

Substituting:

$$I_n = W \cdot \frac{\sigma a \cdot W_{00}}{3L}\left[3(d^2 - s^2) - 2(d^3 - s^3)\right] \tag{3.23}$$

$$I_n = W \cdot \beta W_{00}^2\left[3(d^2 - s^2) - 2(d^3 - s^3)\right] \tag{3.24}$$

Equation 3.24 is Shockley's expression[6] for drain current in the linear region. When the device enters saturation, one end is pinched off(normally the drain). Thus $d = 1$ and one may derive the equation for the saturation region:

$$I_{sat} = \beta W_{00}^2(1 - 3s^2 + 2s^3) \tag{3.25}$$
$$g_m = 3\beta W_{00}(s - 1) \tag{3.26}$$
$$G_{DS} = 3\beta W_{00}(1 - d) \tag{3.27}$$

## 3.1 Simpler Model

$$I_{ds} = \frac{3}{2}\beta W_{00}^2\left[\frac{(V_{gs} - v_{to})^2}{W_{00}^2} - \frac{(V_{gd} - v_{to})^2}{W_{00}^2}\right] \tag{3.28}$$
$$g_m = 3\beta W_{00}(V_{gd} - V_{to}) \tag{3.29}$$
$$G_{ds} = 3\beta W_{00}(V_{gd} - V_{to}) \tag{3.30}$$

**General power law:** It was found that a general power law provided a better fit for real devices[5].

$$I_{ds} = \beta\big[(V_{gs} - V_{to})^Q - (V_{gd} - V_{to})^Q\big] \tag{3.31}$$

Where $Q$ is dependent on the doping profile and a good fit is usually obtained for Q between 1.5 and 3. A general power low is approximately equal to Shockley's equation for Q=2.4. $\beta$ is also empirically chosen and is proportion to the previous $\beta$

$$\beta \propto \frac{\sigma a W}{3LW_{00}} \tag{3.32}$$

Modelling the various regions is done though model binning. This however infers that a sharp transition exists from one region to another, which may not be accurate.

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_{to} \\ \beta\big[(V_{gs} - V_{to})^Q - (V_{gd} - V_{to})^Q\big] & V_{gs} \leq V_{gd} \\ \beta(V_{gs} - V_{to})^Q & V_{gs} > V_{gd} \end{cases} \tag{3.33}$$

# Chapter 4

# Large Signal Models

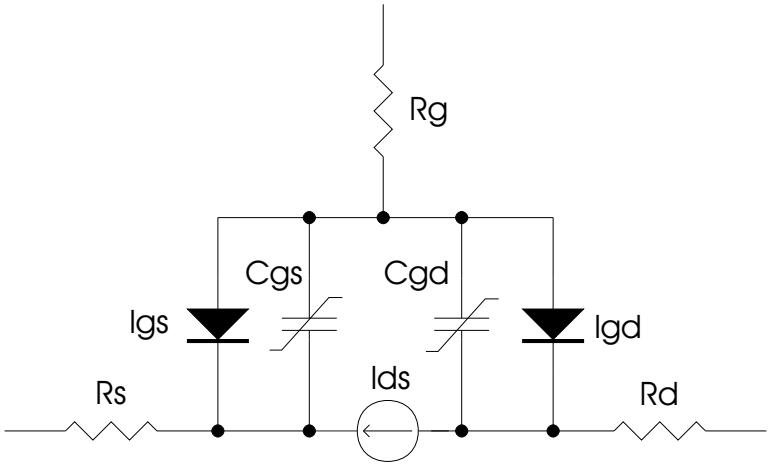


Figure 4.1: Large Signal Model

A large signal model is shown in figure 4.1. Extrinsic elements $R_g$, $R_d$ and $R_s$ must be de-imbed and are independent of bias. The gate currents $I_{gs}$ and $I_{gd}$ are modelled with diode equations as these are simply Schottky contacts[2].

$$i_{GD} = I_s[e^{V_{GD}/\eta V_t} - 1] - I_{BD}[e^{-V_{GD}/V_{BD}} - 1] \tag{4.1}$$

$$i_{GS} = I_s[e^{V_{GS}/\eta V_t} - 1] - I_{BD}[e^{-V_{GS}/V_{BD}} - 1] \tag{4.2}$$

$I_{BD}$ models the reverse breakdown current.

## 4.1 drain model

The DC characteristics for a MESFET are shown in figure 4.2. This is a non-linear function of gate and drain voltage, $V_{gs}$ and $V_{ds}$. Each operating region of the transistor is modelled with some function, pinch-off, linear and saturation as shown in 3.33. A empirical large signal model implements equations which provide a good fit to measured data, converge quickly and a smooth transition between operating regions.

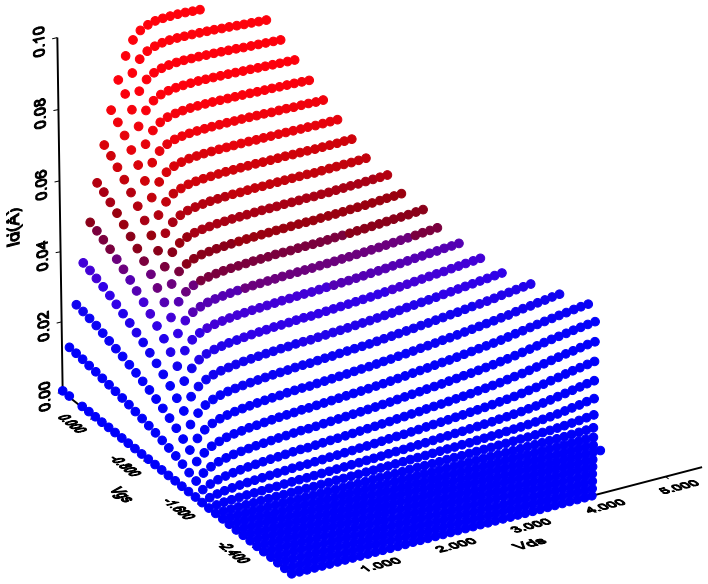


Figure 4.2: MESFET

### 4.1.1 Curtice Model

One of the first Large signal models was proposed by Van Tuyl and Liechti[8]. This was later refined by Curtice[3], which became the basis for many models. Curtice modelled the transition from the Linear to saturation region with a hyperbolic TAN function.

There are many models based on the Curtice model. Most are implemented with the hyperbolic tan function or similar variant. The Curtice model is[3]:

$$I_{ds}(V_{gs}, V_{ds}) = \beta(V_{gs} - V_{TO})^2(1 + \lambda V_{ds})tanh(\alpha V_{ds}). \tag{4.3}$$

where $\beta$ is empirically chosen. $V_{to}$ is the threshold voltage, $\lambda$ the output conductance in saturation region and $\alpha$ determines saturation region.

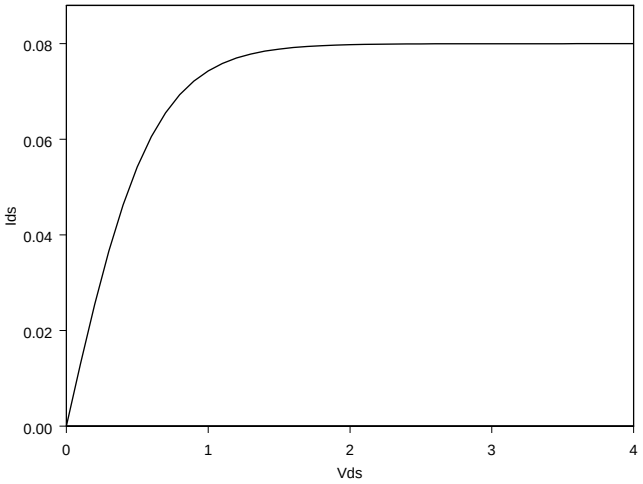


Figure 4.3: TANH function

## 4.2 PS Model

The derivatives of the TANH function differ significantly from those observed in real transistors. This is shown in [2]. Thus a smoothing function appears to be a better implementation. A transistor enters the saturation region when $V_{ds} > V_{sat}$. Thus $V_{ds}$ goes to $V_{sat}$ in the saturation region and is modelled with the following smoothing function.

$$f_{min}(x,y) = \frac{1}{2}((x+y) - \sqrt{(x-y)^2 + \Delta^2}) \tag{4.4}$$

$$f_{max}(x,y) = \frac{1}{2}((x+y) + \sqrt{(x-y)^2 + \Delta^2}) \tag{4.5}$$

$$F(V_{ds}, V_{sat}) = \frac{1}{2}\big[f_{min}(V_{ds}, V_{sat}) - f_{max}(-V_{ds}, -V_{sat})\big] \tag{4.6}$$

Where $\Delta$ determines the degree of smoothing. Thus one may choose or fit $\Delta$ such that the higher derivative are consistent with measured results.

# Chapter 5

# Capacitance Model

Figure 5.1 is an example of a basic capacitor model. This is a poor model and to understand why, one must look at the charge causing the capacitance.

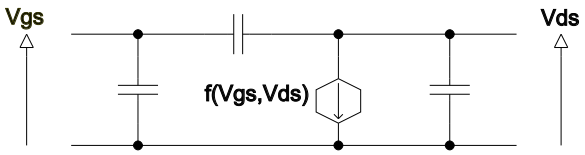


Figure 5.1: Basic Capacitor Model

The capacitance in a MESFET is some function of the charge within the device. Charge in the gate may be balanced by charge in the source or charge in the drain, it thus becomes a significant problem to ensure conservation of charge. To overcome this difficulty, a charge based model is used. Capacitance is particularly useful as it is easily measured. The charge and capacitance are related by:

$$C = \frac{dQ}{dV} \tag{5.1}$$

Where C is the capacitance and Q is the charge. In a three terminal device such as a MESFET there is significant charge in the gate. This is balanced by charge in the channel. Therefore one need only know the charge in the gate to give the relation:

$$Q_G = -Q_S - Q_D \tag{5.2}$$

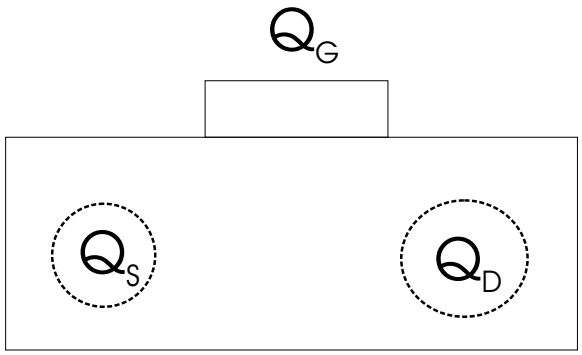


Figure 5.2: Charge Distribution

How this charge is partitioned is unknown at this stage of the analysis. A graphical representation of the charge is shown in fig 5.2. Therefore we have a capacitance due to the charge in the in the gate, source and drain, which may be seen in 5.3. One may note that ordinarily $C_{gs} = C_{sg}$ and $C_{gd} = C_{dg}$ however this is not strictly true as charge in the channel is free to move from source to drain and visa-versa.

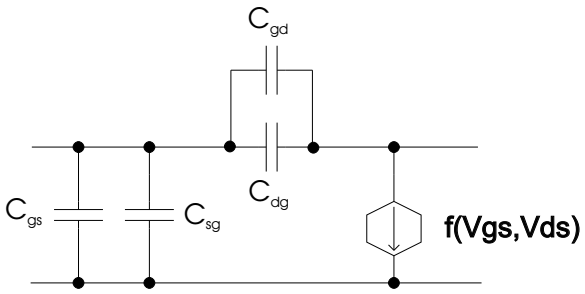


Figure 5.3: Charge Model

The charge equations for figure 5.3:

$$\begin{aligned} C_{gs} &= \frac{\partial Q_g}{\partial V_{gs}} \qquad C_{gd} = \frac{\partial Q_g}{\partial V_{gd}} \\ C_{sg} &= \frac{\partial Q_s}{\partial V_{sg}} \qquad C_{dg} = \frac{\partial Q_d}{\partial V_{dg}} \end{aligned} \tag{5.3}$$

If one assumes the source as the reference, one may calculate the currents.

$$i = \frac{dQ}{dt} \tag{5.4}$$

$$i = \frac{dQ}{dv}\frac{dv}{dt} \tag{5.5}$$

$$I = \left[\frac{dQ}{dv}\right]\left[\frac{dv}{dt}\right] \tag{5.6}$$

$$\begin{bmatrix} i_g \\ i_s \\ i_d \end{bmatrix} = \begin{bmatrix} \frac{\partial Q_g}{\partial V_g} & \frac{\partial Q_g}{\partial V_s} & \frac{\partial Q_g}{\partial V_d} \\ \frac{\partial Q_s}{\partial V_g} & \frac{\partial Q_s}{\partial V_s} & \frac{\partial Q_s}{\partial V_d} \\ \frac{\partial Q_d}{\partial V_g} & \frac{\partial Q_d}{\partial V_s} & \frac{\partial Q_d}{\partial V_d} \end{bmatrix} \begin{bmatrix} \frac{\partial V_g}{\partial t} \\ \frac{\partial V_s}{\partial t} \\ \frac{\partial V_d}{\partial t} \end{bmatrix}$$

If one assumes the quasi-static assumption of Charge conservation. Then any charge in the gate must be balanced by a charge in the channel, thus we get the equation:

$$Q_g + Q_s + Q_d = 0 \tag{5.7}$$

Thus the center row and column go to zero and the matrix reduces to:

$$\begin{bmatrix} i_g \\ i_d \end{bmatrix} = \begin{bmatrix} \frac{\partial Q_g}{\partial V_g} & \frac{\partial Q_g}{\partial V_d} \\ \frac{\partial Q_d}{\partial V_g} & \frac{\partial Q_d}{\partial V_d} \end{bmatrix} \begin{bmatrix} \frac{\partial V_g}{\partial t} \\ \frac{\partial V_d}{\partial t} \end{bmatrix}$$

This result is simply extracted from the imaginary component of network measurements giving:

$$\begin{bmatrix} i_{gs} \\ i_{ds} \end{bmatrix} = j\omega \begin{bmatrix} \frac{\partial Q_g}{\partial V_g} & \frac{\partial Q_g}{\partial V_d} \\ \frac{\partial Q_d}{\partial V_g} & \frac{\partial Q_d}{\partial V_d} \end{bmatrix} \begin{bmatrix} v_{gs} \\ v_{ds} \end{bmatrix}$$

$$\begin{bmatrix} i_{gs} \\ i_{ds} \end{bmatrix} = j \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_{gs} \\ v_{ds} \end{bmatrix}$$

Where $Y_{11}$, $Y_{12}$, $Y_{21}$, $Y_{22}$ is the imaginary component of the admittance.

In an active device, $Y_{12} \neq Y_{21}$ therefore the capacitance is not a simple 3 capacitance network. It will inevitably include a fourth capacitance. One method is to model this capacitor as a trans-capacitance.

$$\begin{aligned} I_{GS} &= j\omega C_{gs}V_{gs} + j\omega C_{gd}V_{gs} - j\omega C_{gd}V_{ds} \\ &= j\omega \begin{bmatrix} C_{gs} + C_{gd} & -C_{gd} \end{bmatrix} \begin{bmatrix} V_{gs} \\ V_{ds} \end{bmatrix} \end{aligned} \tag{5.8}$$

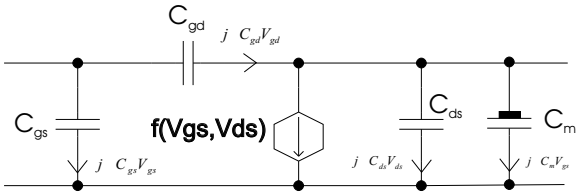


Figure 5.4: Capacitor Model

$$\begin{aligned} I_{DS} &= -j\omega C_{gd}V_{gs} + j\omega C_{gd}V_{ds} + j\omega C_{ds}V_{ds} + j\omega C_m V_{gs} \\ &= j\omega \begin{bmatrix} C_m - C_{gd} & C_{gd} + C_{ds} \end{bmatrix} \begin{bmatrix} V_{gs} \\ V_{ds} \end{bmatrix} \end{aligned} \tag{5.9}$$

$$\begin{bmatrix} i_{gs} \\ i_{ds} \end{bmatrix} = j\omega \begin{bmatrix} C_{gs} + C_{gd} & -C_{gd} \\ C_m - C_{gd} & C_{gd} + C_{ds} \end{bmatrix} \begin{bmatrix} v_{gs} \\ v_{ds} \end{bmatrix}$$

One may combine the transconductance with the trans-capacitance[9]:

$$\text{trans-term} = g_m + j\omega C_m \tag{5.10}$$

$$= g_m e^{-j\omega\tau} \tag{5.11}$$

$$\tau = -\frac{C_m}{g_m} \tag{5.12}$$

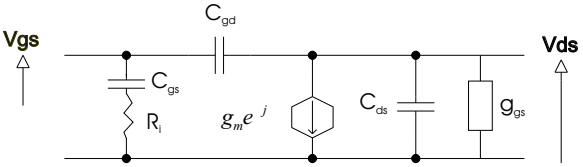


Figure 5.5: Common model

This gives the circuit shown in Figure 5.5. The resistance $R_i$ is added which represents the significant real component of $Y_{11}$ that is observed in MESFETs. Both $R_i$ and $\tau$ will affect the high frequency phase.

# Chapter 6

# Appendix

## 6.1 Definitions

$N_c$ - number energy levels in conduction band
$N_v$ - number energy levels in valence band
$E_d$ - Ionization energy of donor
$m_e$ - effective mass of electron

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